Question Paper Code : 6302

B.C.A. (Semester-I) Examination, 2019

(New Course)

MATHEMATICS-I

[Third Paper (BCA-103)]

Time : Three Hours] **[Maximum Marks : 70**

Note : Answer five questions in all. Question **No. 1** is **compulsory**. Besides this, attempt one question from each unit.

1. Answer the following : [10x3=30]

(a) If A and B are symmetric matries, prove that AB-BA is a skew symmetric matrix.

(b) Transform $\begin{bmatrix} 1 & 3 & 3 \\ 2 & 4 & 10 \\ 3 & 8 & 4 \end{bmatrix}$ into a unit matrix by using elementary transformations.

(c) If $y = \sin 2x \sin 3x$, find y_n

(d) Find the n^{th} derivative of log(ax+b).

(e) If $z = e^{ax+by} f(ax - by)$, show that :

$$b\frac{\partial z}{\partial x} + a\frac{\partial z}{\partial y} = 2abz$$

(f) Find the asymptotes of the curve:

$$x^3 + 2x^2y - xy^2 - 2y^3 + 4y^2 + 2xy + y - 1 = 0$$

(g) Expand a^x and e^x in power of x, by Maclaurin's theorem.

(h) Find the extreme values of function :

$$x^3 + y^3 - 3ax;$$

(i) Find the divergence of the vector field :

$$\overline{V} = x^2y^2\hat{i} + 2xy\hat{j} + (y^2 - xy)\hat{k}$$

(j) For any closed surface S, prove that :

$$\iint_S \text{curl}\overline{F}.\hat{n}\, dS = 0$$

UNIT-I

2. Find the eigenvalues and eigenvectors of the matrix: [10]

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

3. Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence, compute A^{-1}. Also find the matrix represented by :

$$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I. \quad [10]$$

UNIT-II

4. If $y = \cos(m \sin^{-1}x)$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2-m^2)y_n = 0$. [10]

5. If $u = \sin^{-1}\left[\dfrac{x+2y+3z}{\sqrt{x^8+y^8+z^8}}\right]$, show that :

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} + 3\tan u = 0 \quad [10]$$

UNIT-III

6. Find the dimensions of a rectangular box of maximum capacity whose surface area is given when box is closed. [10]

7. Show that $\beta(p,q) = \int_0^1 \frac{x^{p-1}+x^{q-1}}{(1+x)^{p+q}}dx$. [10]

UNIT-IV

8. Find the directional derivative of $\phi = 5x^2y - 5y^2z + \frac{5}{2}z^2x$ at the point P(1,1,1) in the direction of the line $\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$. [10]

9. Use divergence theorem to evaluate the surface integral $\iint_S (xdydz + ydzdx + zdxdy)$ where 'S' is the portion of the plane x + 2y + 3z = 6 which lies in the first octant. [10]

----- X -----